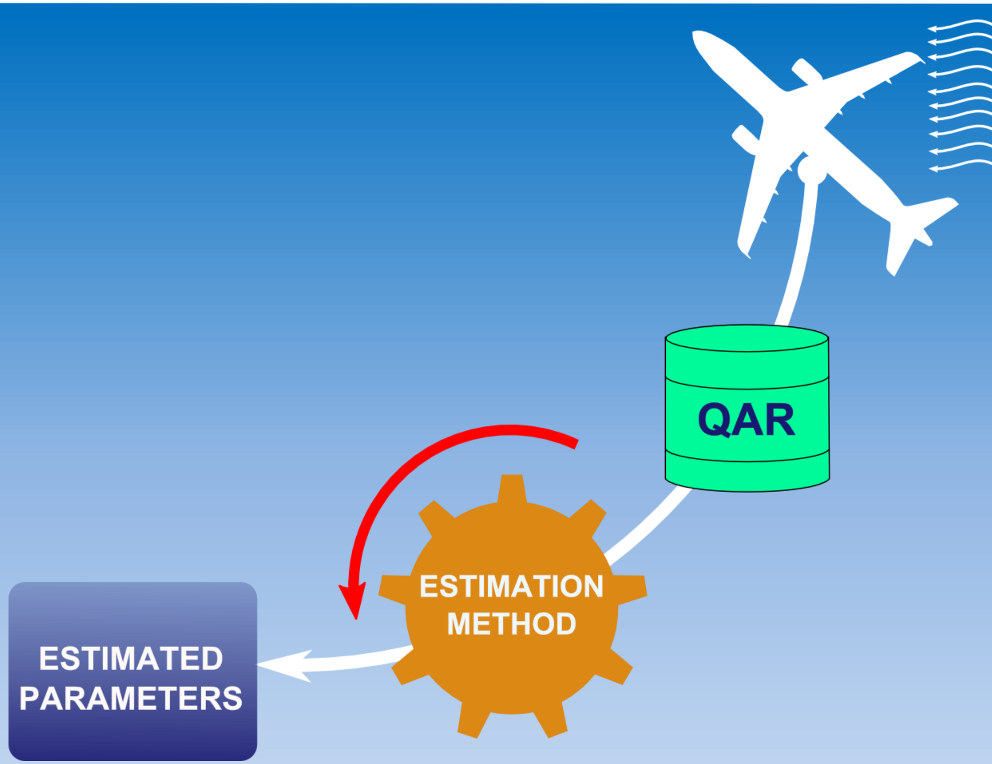

Extracting Unmeasured Parameters Using Estimation Method

.... Looking Deeper into the Data

Javensius Sembiring
PhD Student
Institute of Flight System Dynamics
TUM, Germany

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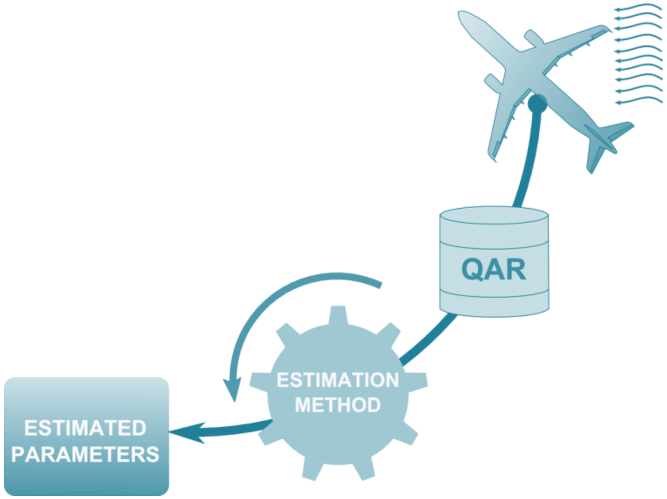
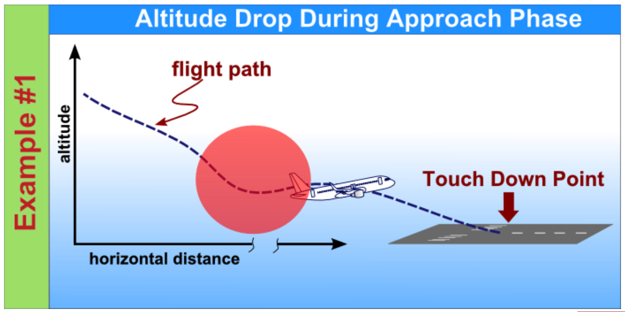
Content



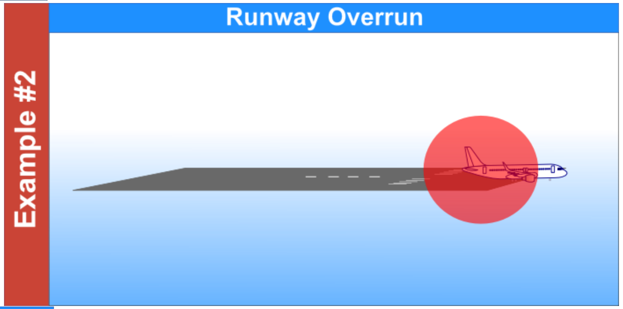
1. Background

- **FDM State of the Art:**

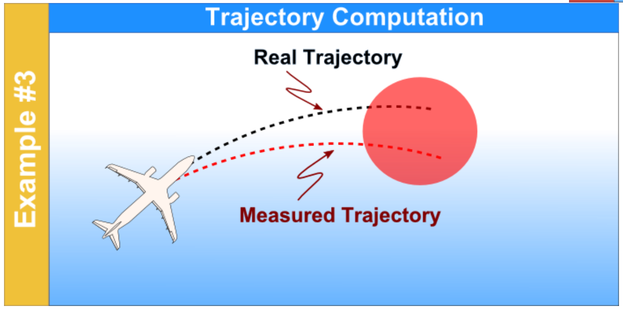
  'Based on recorded variables and analytical analysis'.



- Elevator (jammed or not ?)
- Wind speed – *but we only have horizontal wind speed*.
- ?



- Spoiler deployment.
- Thrust Reverser.
- Brakes.
- ?



- Yaw, pitch, roll angles.
- Latitude, longitude, altitude.
- ?

But how if all the measured variables *do not provide us with sufficient information* which enable us to investigate the cause of the event (or the worst case might be an incident/accident)

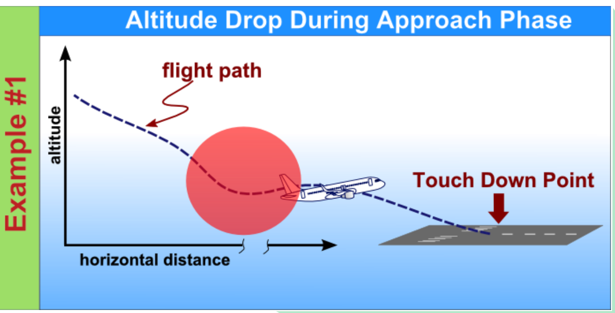
Image courtesy : www.npr.org

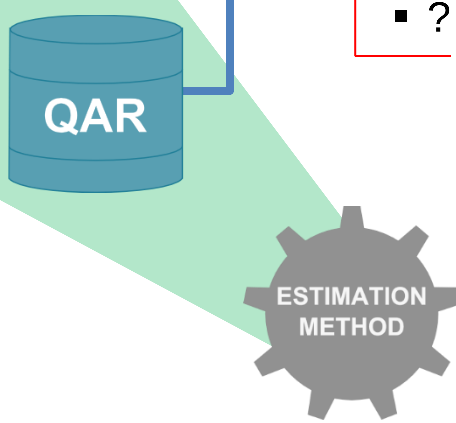
One answer might be ....

We need to look deeper into the data!

1. Background

- **Beyond FDM State of the Art:**
  - Extracting parameters which are not measured on QAR data.
  - These unmeasured parameters provide information which can be used for event detection or incident/accident investigation.





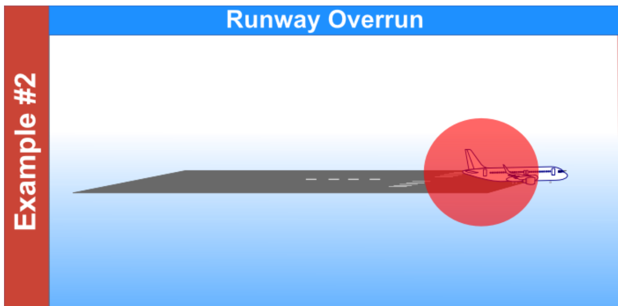
Some investigated parameters based on current FDM approach.

- Elevator (jammed or not ?)
- Wind speed – but we only have horizontal wind speed.
- ?

More parameters are revealed in order to provide more information

- Lift and drag coefficient
- Wind speed component (x,y,z)

- **Beyond FDM State of the Art**



Some investigated parameters based on current FDM approach.

- Spoiler deployment.
- Thrust Reverser.
- Brakes.
- ?

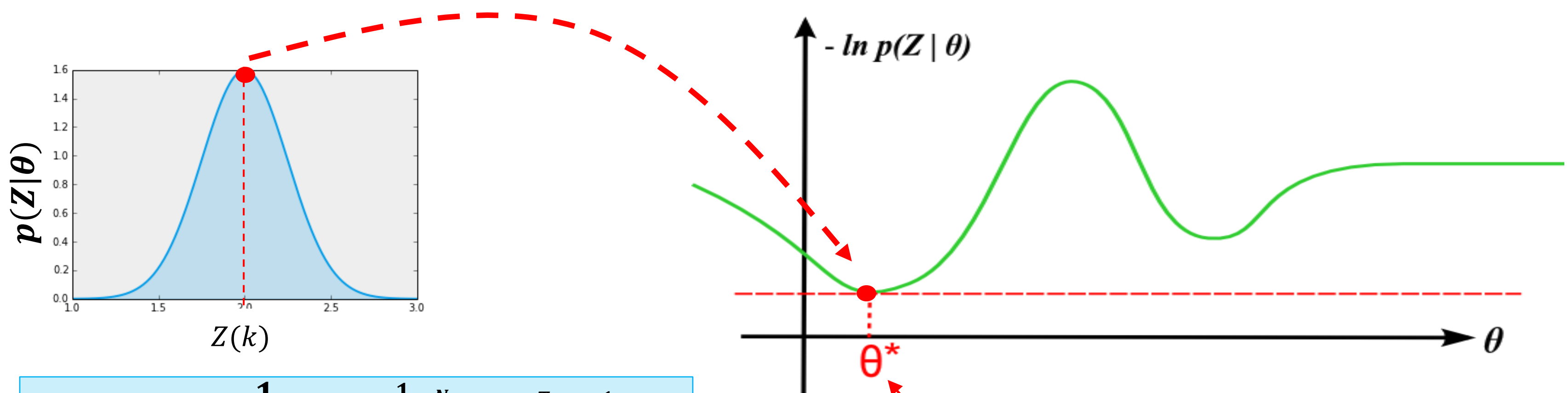
More parameters are revealed in order to provide more information

- Runway friction coefficient

2. Technique Used in Flight System Identification

- The technique called Output Error Method is used for estimating the unmeasured/unrecorded parameters.
- This technique is commonly used in Flight Vehicle System Identification.
- The Output Error Method works based on Maximum Likelihood principle.

‘ Select parameters θ which maximize the conditional probability of measurement (Z) given parameters (θ) or which minimizes the $-\ln\left(p(Z|\theta)\right)$’



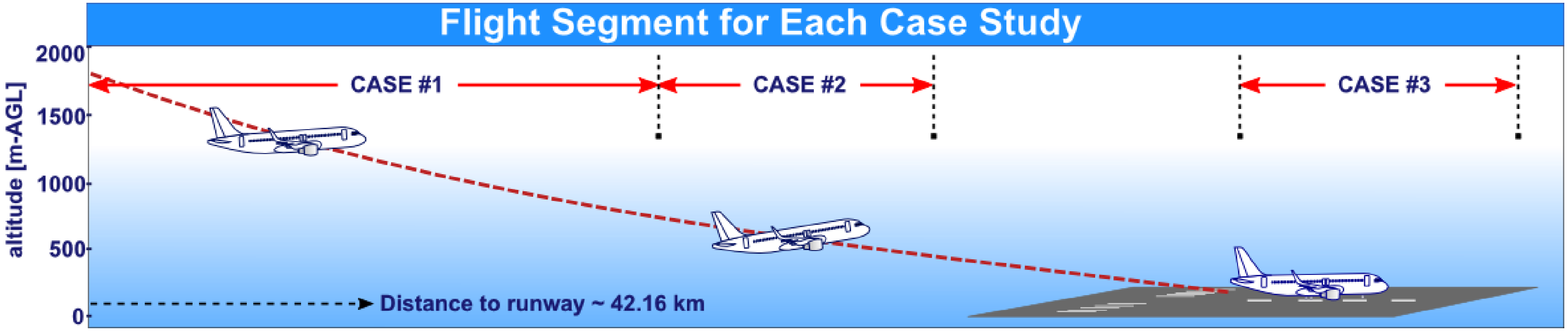
which minimizes the $-\ln p\,(Z|\theta)$

$$p(z|\theta) = \frac{1}{(2\pi)^{\frac{n_y}{2}}\sqrt{|R|}}\, e^{-\frac{1}{2}\sum_{k=1}^{N}[z-y]^T[R]^{-1}[z-y]}$$

Z = measurement
y = model -- function of θ
R = measurement covariance matrix

3. Case Study

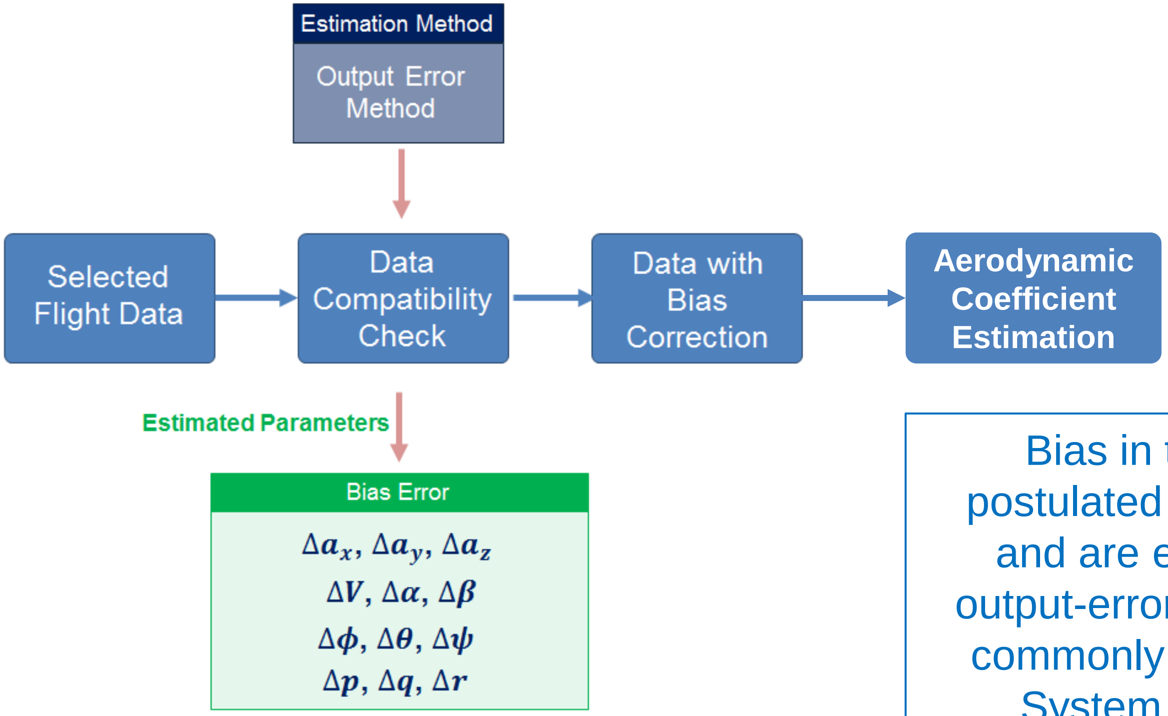
- **Cases:**
  - Case #1 → Lift & drag coefficient due to flap deflection.
  - Case #2 → Wind speed component estimation.
  - Case #3 → Runway friction and aerodynamic coefficient.



- **Data used: A340-600 QAR Data. Variables include:**
  - Measurement from accelerometers (a_x, a_y, a_z)
  - Measurement from gyroscope (yaw, pitch, roll, and their rate).
  - Measurement from pitot tube, α and β- vane (airspeed, AoA, AoS).
  - Measurement from altimeter (altitude).
- **In every segment, the bias in the measurement is estimated in order to improve the accuracy of the estimates.**

- **Case #1: Lift & Drag Coefficient Increment due to Flap Deflection**

Estimation Step Workflow



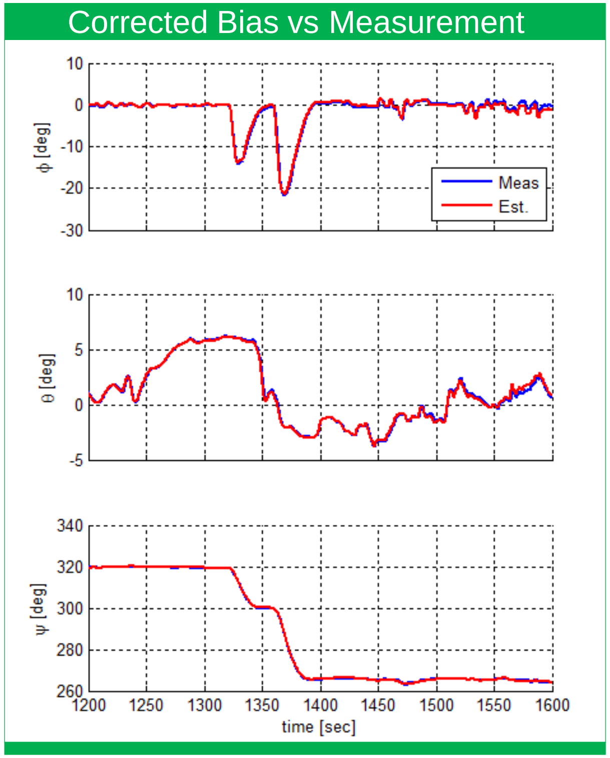
Bias in the measurement are postulated in **kinematic equations** and are estimated by employing output-error method. This process is commonly applied in Flight Vehicle System Identification for **data compatibility check** purpose.

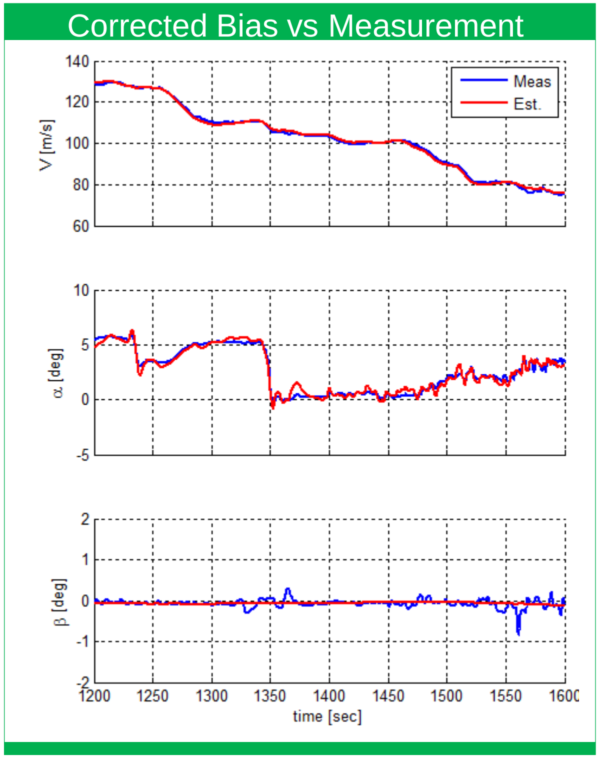
- $\Delta a_{x,y,z}$: bias in accelerometer sensor.
- $\Delta V, \Delta\alpha, \Delta\beta$: bias in flow measurement.
- $\boldsymbol{\Delta\phi}, \boldsymbol{\Delta\theta}, \boldsymbol{\Delta\varphi}, \boldsymbol{\Delta p}, \boldsymbol{\Delta q}, \boldsymbol{\Delta r}$: bias in gyroscope sensors.

3. Implementation and Results

- **Case #1: Lift & Drag Coefficient Increment due to Flap Deflection**

Bias Estimation Results





Parameters	Value		Std.Dev.
Δa_x:	-7.027E-03	[m/s²]	7.085E-04
Δa_y:	-2.392E-01	[m/s²]	7.671E-04
Δa_z:	2.021E-01	[m/s²]	7.671E-05
Δp:	-5.261E-04	[rad/s]	6.268E-07
Δq:	2.869E-06	[rad/s]	5.930E-07
Δr:	-5.969E-04	[rad/s]	8.718E-07

Parameters	Value		Std.Dev.
ΔV:	1.305E-01	[m/s]	4.572E-02
$\Delta \alpha$:	-1.527E-02	[rad]	3.243E-04
$\Delta \beta$:	-1.398E-03	[rad]	3.373E-05
$\Delta \phi$:	5.236E-03	[rad]	2.184E-04
$\Delta \theta$:	-1.057E-03	[rad]	8.299E-05
$\Delta \varphi$:	-1.141E-02	[rad]	2.009E-04

- **Case #1: Lift & Drag Coefficient Increment due to Flap Deflection**

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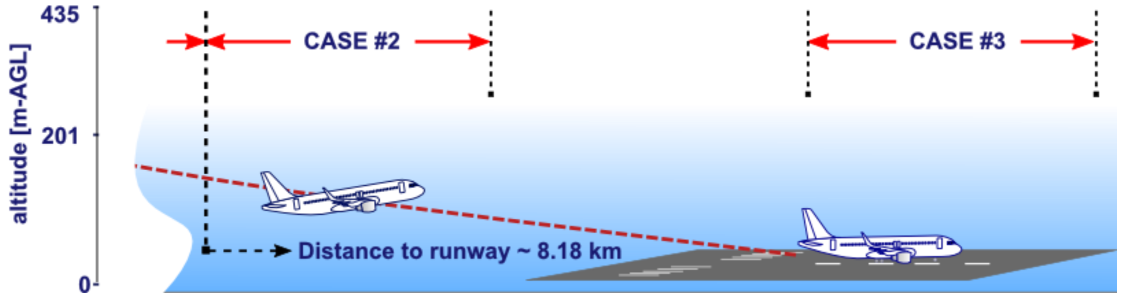
Case #2: Wind Speed Component Estimation – Postulated Model



data length = 100 sec.

$$V_w^E = C_b^E \boldsymbol{V_b} - C_b^E C_a^b \boldsymbol{V_a}$$

V_a = Airspeed in aerodynamic frame.

V_b = Speed in body frame.

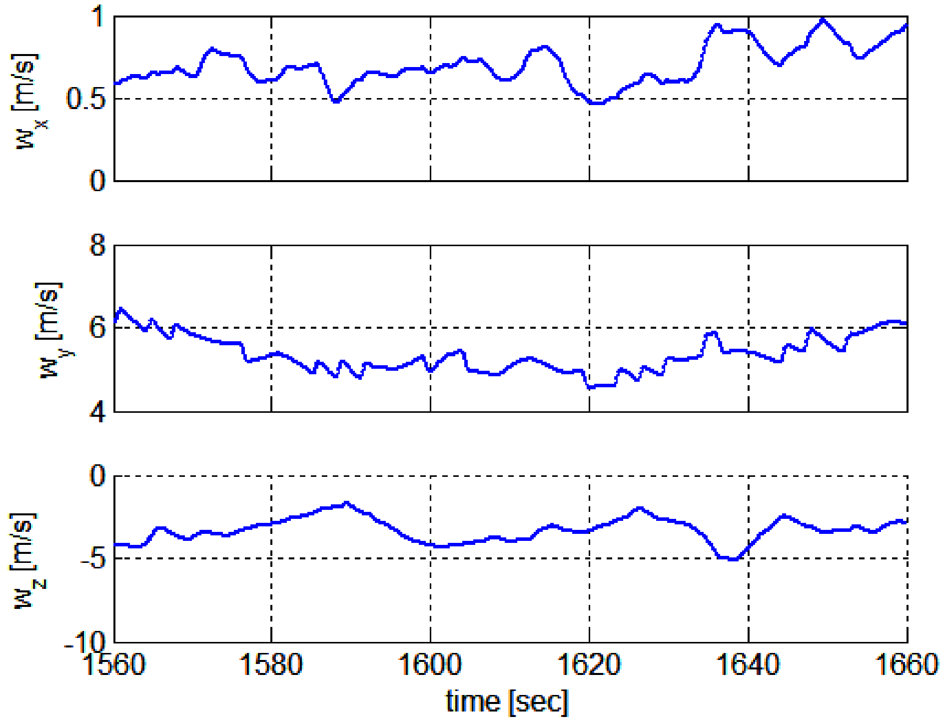
V_w^E = wind speed component earth fame.

C_b^E = matrix transformation from body to earth frame.

C_a^b = matrix transformation from wind to body frame.

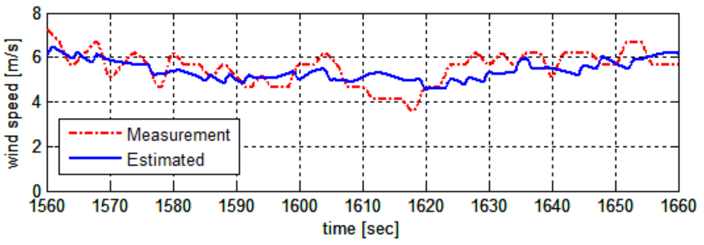
- **Case #2: Wind Speed Component Estimation (Results)**

Wind Speed Component Estimation



Horizontal Wind Speed [Estimated vs Measurement]

$$w_{hor} = \sqrt{w_x^2 + w_y^2}$$



R^2 = 83 %

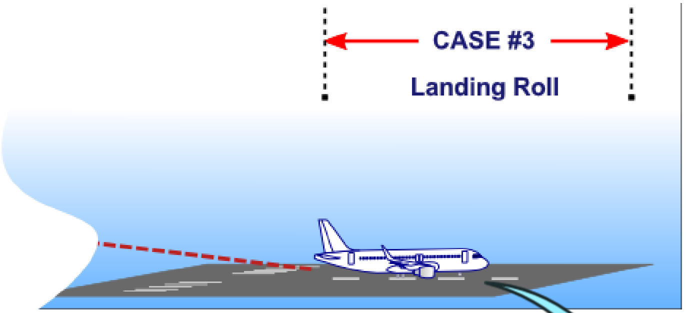
R^2 = coefficient of determination

These components are not recorded on QAR Data

Proof of concept – comparison between measured horizontal wind speed and reconstructed horizontal wind speed.

3. Implementation and Results

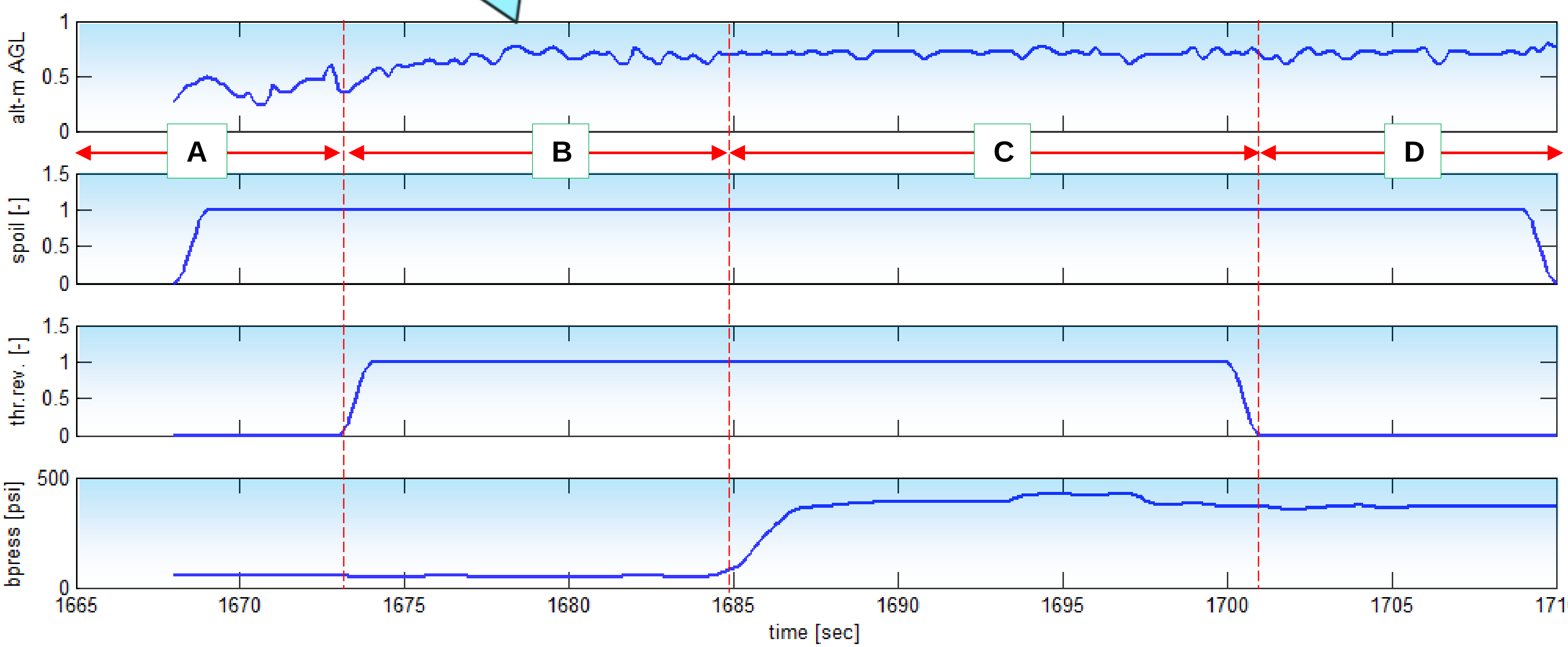
- **Case #3: Runway Friction, Lift and Drag Coefficient Estimation** **(Data Selection)**



data length = 45 sec.

Landing Roll Segmentation

- **A**: Spoiler Deployed
- **B**: Spoiler + Thrust Reverser Deployed
- **C**: Spoiler + Thrust Reverser Deployed + Braking
- **D**: Spoiler Deployed + Braking



- **Case #3: Runway Friction, Lift and Drag Coefficient Estimation** **(Postulated Model)**

Phase A: Spoiler Deployed

$$\mathbf{m}\cdot\hat{a}_x \approx -\bar{q}\cdot S\cdot C_{D,S}-\mu_{roll}\cdot\left(m\cdot g-\bar{q}\cdot S\cdot C_{L,G}\right)+\delta_T T_0$$

Phase B: Spoiler + Thrust Reverser Deployed

$$\mathbf{m}\cdot\hat{a}_x \approx -\bar{q}\cdot S\cdot C_{D,S}-\mu_{roll}\cdot\left(m\cdot g-\bar{q}\cdot S\cdot C_{L,G}\right)-\delta_T T_{0,REV}$$

Phase C: Spoiler + Thrust Reverser Deployed + Braking

$$\mathbf{m}\cdot\hat{a}_x \approx -\bar{q}\cdot S\cdot C_{D,S}-\mu_{rollbrk}\cdot\left(m\cdot g-\bar{q}\cdot S\cdot C_{L,G}\right)-\delta_T T_{0,REV}$$

Phase D: Spoiler Deployed + Braking

$$\mathbf{m}\cdot\hat{a}_x \approx -\bar{q}\cdot S\cdot C_{D,S}-\mu_{rollbrk}\cdot\left(m\cdot g-\bar{q}\cdot S\cdot C_{L,G}\right)-\delta_T T_0$$

Corrected acceleration is obtained from previous step (bias correction) and each equation is then solved by using least square method

$$\boxed{param = [C_{D,S}, \mu_{roll}, \mu_{rollbrk}, C_{L,G}, T_0]}$$

← Parameters to be estimated.

$\hat{a}_x$	: corrected horizontal acceleration	S	: wing area
m	: mass	μ_{roll}	: rolling friction coef.
$\bar{q}$	: dynamic pressure	$\mu_{rollbrk}$	: rolling+braking friction coef.
g	: gravity constant	$C_{L,G}$	: lift coef. during ground run.
$C_{D,S}$	: drag coef. during spoiler deployed.	T_0	: thrust
δ_T	: throttle input		

- **Case #3: Runway Friction, Lift and Drag Coefficient Estimation** **(Results)**

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5. Conclusions

- Estimation technique, commonly applied in System Identification field, can also be implemented on QAR data to estimate the unmeasured/unrecorded parameters.

- The extracted parameters can be used for event detection or for investigating the cause of incident/accident in which the measured variables on QAR data are not able to provide such information.

- The estimation technique described before can be integrated into the current FDM software aiming at extending the FDM capabilities – it would provide more information (give accurate analysis).

5. Conclusions

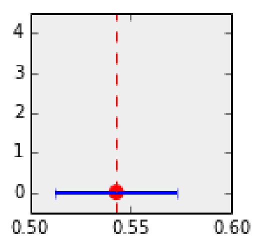
- The computation time took not more than 30 seconds for each flight phase. The computation time is highly depending on the number of measurements involved in the estimation process.

- Can be run on massive data but requires an algoritm for flight segmentation.

Recent Results

- It would be better to represent the parameters in distribution form instead of a single value. This is due to some contributing factors which makes QAR data has some uncertainties such as:
  - Untailored flight manuever.
  - Low and different sampling rate.
  - Untailored control input.

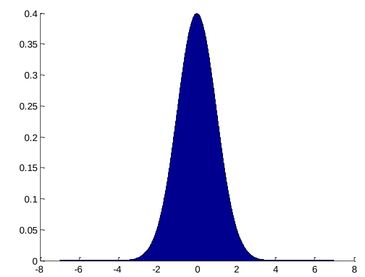
Point Estimate



$\theta =$ single value

vs

Bayesian Method



$\theta =$ distributed

Parameter distribution captures the uncertaintes in the data.

Lift Coefficient and Its Increment Due to Flap Deflection in Distribution Form

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Thank you!

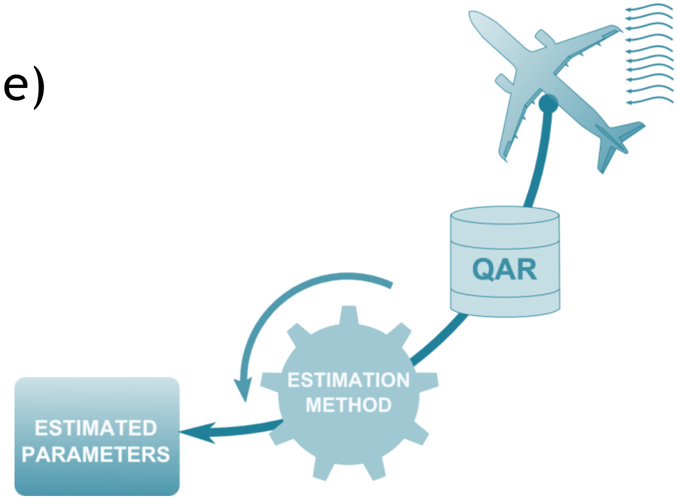
Professor

Florian Holzapfel (florian.holzapfel@tum.de)

PhD Students

Javensius Sembiring (javensius.sembiring@tum.de)
Ludwig Drees (ludwig.drees@tum.de)
Lukas Höhndorf (lukas.hoehndorf@tum.de)
Chong Wang (chong.wang@tum.de)

Institute of Flight System Dynamics
Technische Universität München
www.fsd.mw.tum.de



Additional Slides

Maximum Likelihood Principle

$$\max \rightarrow p(z|\theta) = \frac{1}{(2\pi)^{\frac{n_y}{2}}\sqrt{|R|}} e^{-\frac{1}{2}\sum_{k=1}^{N}[z(t_k)-y(t_k)]^T[R]^{-1}[z(t_k)-y(t_k)]} \quad \dots (1)$$

or minimizing the – logarithmic of likelihood function:

$$J(\Theta) = (L(z|\Theta) = -\frac{1}{2}\sum_{k=1}^{N}[z(k)-y(k)]^T[R]^{-1}[z(k)-y(k)] + \frac{N}{2}ln[det(R)] + \frac{Nn_y}{2}ln(2\pi) \quad \dots (2)$$

$$\frac{\partial J(\Theta)}{\partial \Theta} = 0 \longrightarrow \left(\frac{\partial J(\Theta)}{\partial \Theta}\right)_{i+1} \approx \left(\frac{\partial J(\Theta)}{\partial \Theta}\right)_i + \left(\frac{\partial^2 J(\Theta)}{\partial \Theta^2}\right)_i \Delta\Theta = 0 \quad \dots (3)$$

$$\Delta\Theta = -\left[+\left(\frac{\partial^2 J(\Theta)}{\partial \Theta^2}\right)_i\right]^{-1}\left(\frac{\partial J(\Theta)}{\partial \Theta}\right)_i \quad \dots (4)$$

$$\Delta\Theta = -\mathcal{F}^{-1}\mathcal{G} \quad \dots (5)$$

where:

$$\mathcal{F} = \sum_{k+1}^{N}\left[\frac{\partial y(t_k)}{\partial \Theta}\right]^T R^{-1}\left[\frac{\partial y(t_k)}{\partial \Theta}\right]$$

$$\mathcal{G} = -\sum_{k+1}^{N}\left[\frac{\partial y(t_k)}{\partial \Theta}\right]^T R^{-1}[z(t_k) - y(t_k)]$$

z = measurement/observation

y = output model

n_y = number of output

R = matrix covariance

Parameters are updated by using equation (6) below:

$$\Theta_{i+1} = \Theta_i + \Delta\Theta \quad \dots (6)$$

Kinematic Equations

- Kinematic equations are used for data compatibilty check aiming at determining the bias error or scale factor which might exist in the measurements.

State Equations:

$$\dot{V} = (\hat{a}_x cos\alpha + \hat{a}_z sin\alpha)cos\beta + \hat{a}_y sin\beta + g(cos\theta cos\phi sin\alpha cos\beta + cos\theta sin\phi cos\beta - sin\theta cos\alpha cos\beta)$$

$$\dot{\alpha} = \frac{1}{V cos\beta}[\hat{a}_z cos\alpha - \hat{a}_x sin\alpha + g(cos\theta cos\phi cos\alpha + sin\theta sin\alpha)] + \hat{q} - tan\beta(\hat{p} cos\alpha + \hat{r} sin\alpha) \quad (1)$$

$$\dot{\beta} = \frac{1}{V}[\hat{a}_y cos\beta - (\hat{a}_x cos\alpha + \hat{a}_z sin\alpha)sin\beta + g(cos\theta sin\phi cos\beta + (sin\theta cos\alpha - cos\theta cos\phi sin\alpha)sin\beta)] + \hat{p} sin\alpha - \hat{r} cos\alpha$$

$$\dot{\phi} = \hat{p} + \hat{q}\sin\phi\tan\theta + \hat{r}\cos\phi\tan\theta$$

$$\dot{\theta} = \hat{q} cos\phi - \hat{r}\sin\phi$$

$$\dot{\psi} = \hat{q} sin\phi sec\theta + \hat{r} cos\phi sec\theta$$

where,

$$\hat{a}_x = a_{xm} - \Delta a_x \qquad \hat{p} = p_m - \Delta p$$
$$\hat{a}_y = a_{ym} - \Delta a_y \qquad \hat{q} = q_m - \Delta q$$
$$\hat{a}_z = a_{zm} - \Delta a_z \qquad \hat{r} = r_m - \Delta r$$

Measurement Equations:

$$V_m = V + \Delta V \qquad \phi_m = \phi + \Delta\phi$$
$$\alpha_m = K_\alpha \alpha + \Delta\alpha \qquad \theta_m = \theta + \Delta\theta \quad (2)$$
$$\beta_m = K_\beta \beta + \Delta\beta \qquad \psi_m = \psi + \Delta\psi$$

Parameters to be Estimated:

$$\Theta = [\Delta a_x \; \Delta a_y \; \Delta a_z \; \Delta\phi \; \Delta\theta \; \Delta\psi \; \Delta p \; \Delta q \; \Delta r \; \Delta V \; \Delta\alpha \; \Delta\beta \; K_\alpha \; K_\beta]^T \quad (3)$$

Postulated Model for Aerodynamic Estimation

aerodynamic model

$$\hat{a}_x \approx \frac{1}{mg}(\bar{q}SC_X + T)$$
$$\hat{a}_z \approx \frac{1}{mg}(\bar{q}SC_Z)$$

... in body axes (1)

$$C_L = C_{L_0} + \Delta C_L$$
$$C_D = C_{D_0} + \Delta C_D$$

(3)

$\hat{a}_x, \hat{a}_z$, and $\hat{\alpha}$ are obtained from Data Compatibility Check.

$$C_L = -C_Z cos\hat{\alpha} + C_X sin\hat{\alpha}$$
$$C_D = -C_X cos\hat{\alpha} - C_Z sin\hat{\alpha}$$

... in wind axes (2)